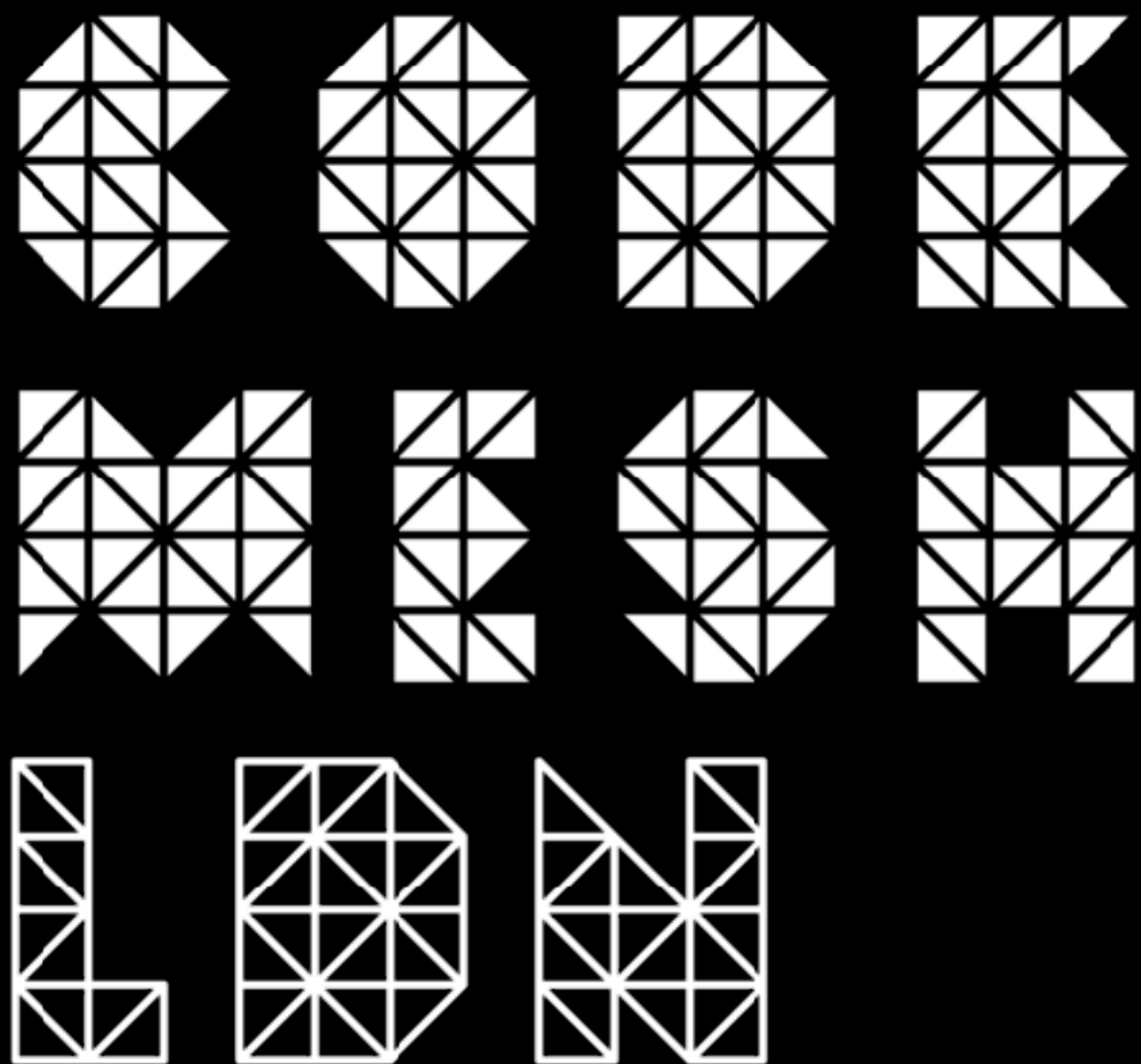


Quantitative program reasoning in Granule via graded modal types



Dominic Orchard



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The Granule Project

<https://granule-project.github.io/>

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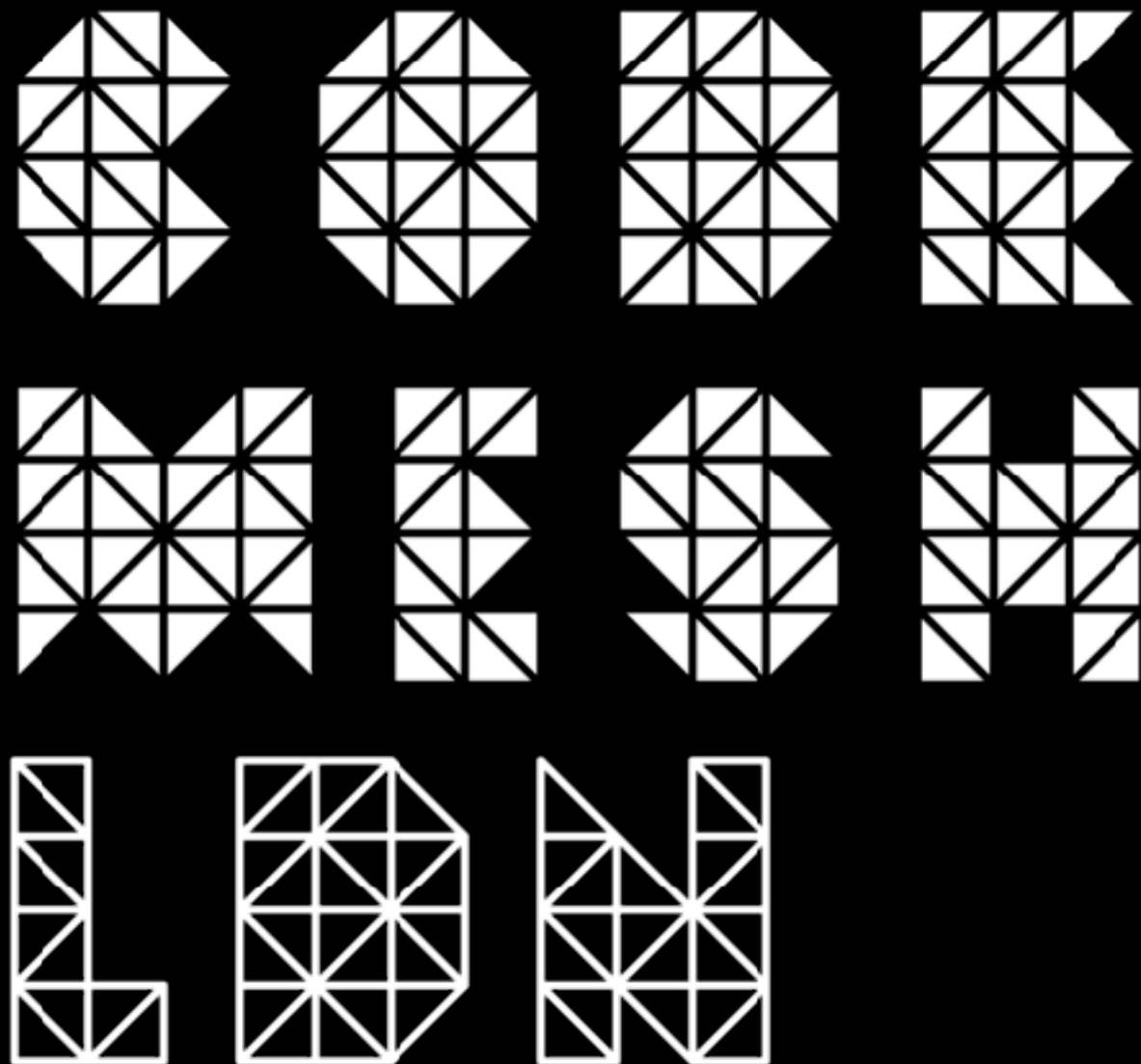
Jack Hughes

Great PhD students!



ALAN PERLIS AMERICAN COMPUTER SCIENTIST

A language that doesn't affect the way you think about programming, is not worth knowing.



Data

~~Infinately copiable~~

~~Arbitrarily discardable~~

~~Universally unconstrained~~

Data

as a resource

Linear types as a basis

LINEAR LOGIC*

1987

Jean-Yves GIRARD

Équipe de Logique Mathématique, UA 753 du CNRS, UER de Mathématiques, Université de Paris VII, 75251 Paris, France

Communicated by M. Nivat
Received October 1986

A la mémoire de Jean van Heijenoort

Abstract. The familiar connective of negation is broken into two operations: linear negation which is the purely negative part of negation and the modality “of course” which has the meaning of a reaffirmation. Following this basic discovery, a completely new approach to the whole area between constructive logics and programmation is initiated.

1990

Linear types can change the world!

Philip Wadler
University of Glasgow*

Abstract

The linear logic of J.-Y. Girard suggests a new type system for functional languages, one which supports operations that “change the world”. Values belonging to a linear type must be used exactly once: like the world, they cannot be duplicated or destroyed. Such values require no reference counting or garbage collection, and safely admit destructive array update. Linear types extend Schmidt’s notion of single threading; provide an alternative to Hudak and Bloss’ update analysis; and offer a practical complement to Lafont and Holmström’s elegant linear languages.

An old canard against functional languages is that they cannot change the world:

□ modality — use any number of times



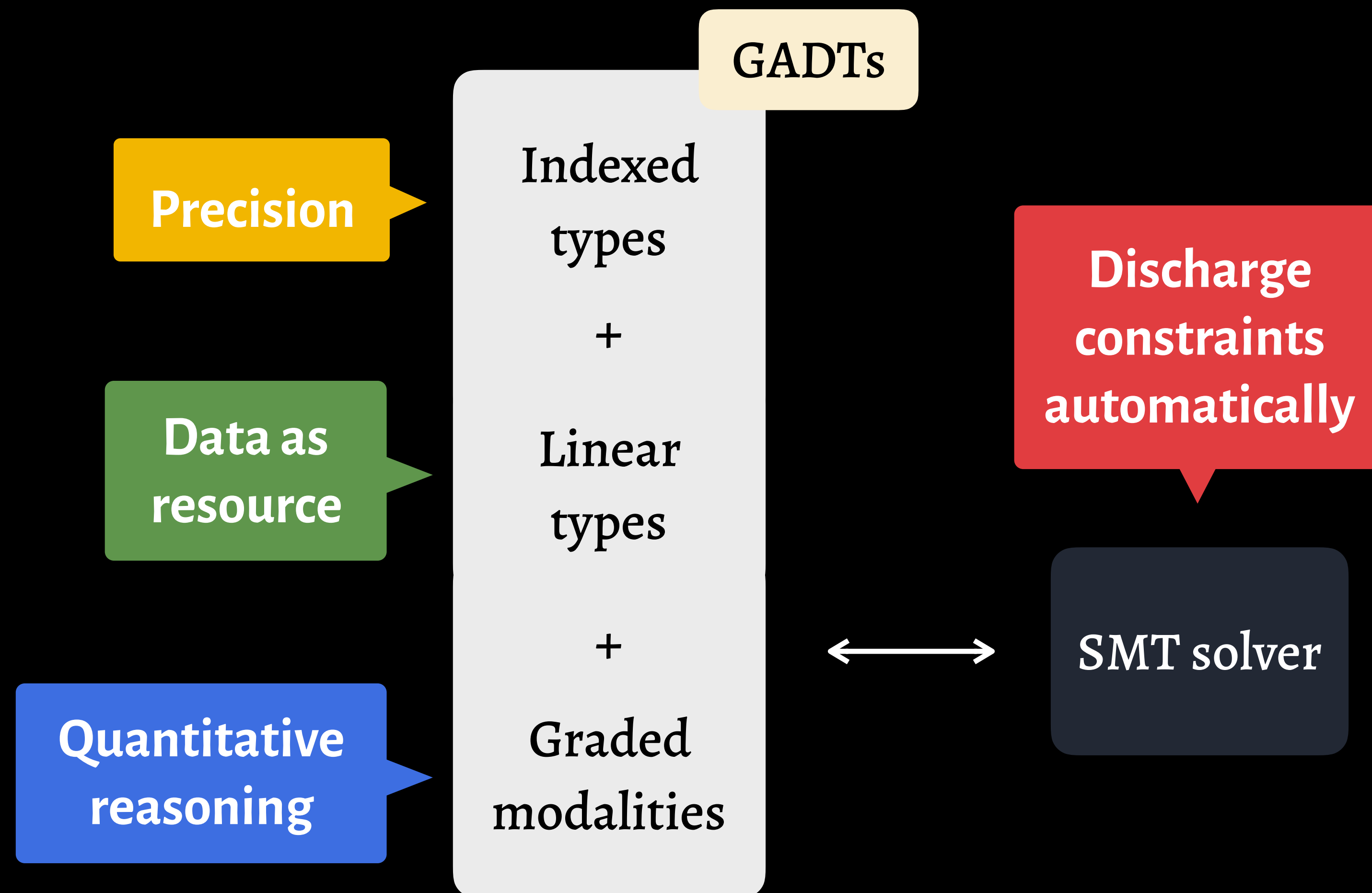
linear types — use exactly once

□ modality — use any number of times

□**n** modality — use **n** number of times

linear types — use exactly once

The **gr**anule language



Graded modalities capture dataflow

Linear



use

1

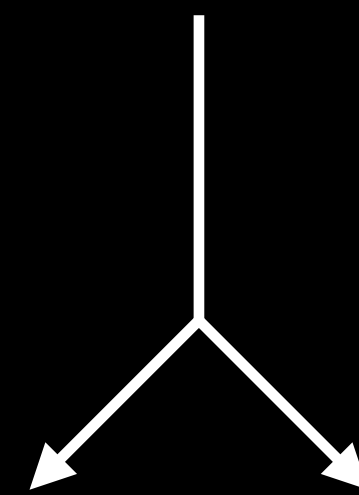
Weaken



discard

0

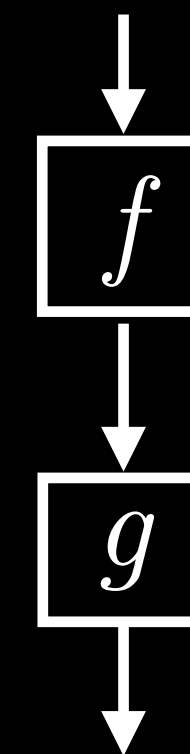
Contract



split

+

Compose



sequence

Grading **algebra** (semiring) captures dataflow

Graded modalities

Two flavours

$A [r]$

Graded "comonads"

Backwards dataflow

$A \langle r \rangle$

Graded "monads"

Forwards dataflow



Precision

Data as resource

Quantitative reasoning

Indexed types

+

Linear types

+

Graded modalities

=

Fine-grained reasoning

about

"Data

As

Resource"

Download and play!

<https://granule-project.github.io/>

1:14

Dominic Orchard, Vilem-Benjamin Liepelt, and Harley Eades III

$$\begin{array}{c}
 \frac{\Sigma \vdash A' \sim A \triangleright \theta_1 \quad \Sigma \vdash \theta_1 B \sim \theta_1 B' \triangleright \theta_2}{\Sigma \vdash A \rightarrow B \sim A' \rightarrow B' \triangleright \theta_1 \uplus \theta_2} U_{\rightarrow} \quad \frac{\Sigma \vdash A \sim A' \triangleright \theta_1 \quad \Sigma \vdash \theta_1 B \sim \theta_1 B' \triangleright \theta_2}{\Sigma \vdash AB \sim A' B' \triangleright \theta_1 \uplus \theta_2} U_{APP} \\
 \frac{(\alpha : \kappa) \in \Sigma}{\Sigma \vdash \alpha \sim \alpha \triangleright \emptyset} U_{VAR=} \quad \frac{\Sigma \vdash A : \kappa \quad (\alpha : \exists \kappa) \in \Sigma}{\Sigma \vdash \alpha \sim A \triangleright \alpha \mapsto A} U_{VAR\exists} \quad \frac{\Sigma \vdash A : \kappa}{\Sigma \vdash A \sim A \triangleright \emptyset} U_{=} \\
 \frac{\Sigma \vdash A \sim A' \triangleright \theta \quad \Sigma \vdash \theta \varepsilon \sim \theta \varepsilon' \triangleright \theta'}{\Sigma \vdash \diamond_{\varepsilon} A \sim \diamond_{\varepsilon'} A' \triangleright \theta \uplus \theta'} U_{\diamond} \quad \frac{\Sigma \vdash A \sim A' \triangleright \theta \quad \Sigma \vdash \theta c \sim \theta c' \triangleright \theta'}{\Sigma \vdash \square_c A \sim \square_{c'} A' \triangleright \theta \uplus \theta'} U_{\square}
 \end{array}$$

Fig. 2. Type unification rules

Type unification is given by relation $\Sigma \vdash A \sim B \triangleright \theta$ (under a context Σ), creating substitutions θ (which has a multi-premise counterpart for $A \sim \alpha$ elided here for brevity). Universally quantified variables can be unified with unification variables via $(U_{VAR\exists})$. In multi-premise unification, subterms are then applied to types being unified in later premises. In programs, we elide the definition of

Quantitative program reasoning with graded modal types
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